Tables

Table 1: Ensembles in statistical mechanics

Ensemble	Partition function	Thermodynamic potential
Microcanonical	$\Delta\Omega(E,V,N)$	$S(E,V,N) = k_B \ln \Delta\Omega(E,V,N)$
Canonical	$Z(T,V,N)$ $= \sum_i \mathrm{e}^{-E_i(V,N)/k_BT}$ $= \int \Delta\Omega(E,V,N)\mathrm{e}^{-E(V,N)/k_BT}dE$ $Y(T,p,N)$	$F(T,V,N) = -k_BT \ln Z(T,V,N)$ $G(T,p,N) = -k_BT \ln Y(T,p,\mu)$
Grand canonical	$\Xi(T,V,\mu)$ $= \sum_N \mathrm{e}^{\mu N/k_BT} Z(T,V,N)$	$J(T,V,\mu) = F - \mu N = -pV$ $= -k_BT \ln \Xi(T,V,\mu)$

Table 2: Thermodynamic functions

Thermodynamic functions (Definition)	Natural variables	Total differential
Entropy S	$\langle E \rangle, V, N$	$\mathrm{d}S = \mathrm{d}\langle E \rangle / T + p\mathrm{d}V/T - \mu \mathrm{d}N/T$
Internal energy $\langle E \rangle$	S, V, N	$\mathrm{d}\langle E \rangle = T\mathrm{d}S - p\mathrm{d}V + \mu \mathrm{d}N$
Enthalpy $H = \langle E \rangle + pV$	S, p, N	$\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}p + \mu \mathrm{d}N$
Helmholtz free energy $F = \langle E \rangle - TS$	T, V, N	$\mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V + \mu \mathrm{d}N$
Gibbs free energy $G = F + pV = N\mu$	T, p, N	$\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}p + \mu \mathrm{d}N$
Grand potential $J = F - G = -pV$	T, V, μ	$\mathrm{d}J = -S\mathrm{d}T - p\mathrm{d}V - N\mathrm{d}\mu$

Table 3: Relations of different thermodynamic quantities to the partition function

Canonical partition function
(constant number of particles)

$$Z(T,V,N) = \int \Delta\Omega(E,V,N)\mathrm{e}^{-\beta E}dE$$

Discrete energy levels:
$Z(T,V,N) = \sum_i \mathrm{e}^{-\beta E_i}$

$\downarrow$

Free energy

$F(T,V,N) = -k_BT\ln Z(T,V,N)$		$G(T,P,N) = F(T,V,N) + PV$	
$\swarrow$	$\searrow$	$\swarrow$	$\searrow$
Pressure	Entropy	Entropy	Volume
$P = -(\frac{\partial F}{\partial V})_T$	$S = -(\frac{\partial F}{\partial T})_V$	$S = -(\frac{\partial G}{\partial T})_P$	$V = (\frac{\partial G}{\partial P})_T$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Inverse compressibility	Heat capacity (constant V)	Heat capacity (constant P)	Isothermal compressibility
$\frac{1}{\kappa_T} = -V(\frac{\partial P}{\partial V})_T$	$C_V = T(\frac{\partial S}{\partial T})_V$	$C_P = T(\frac{\partial S}{\partial T})_P$	$\kappa_T = -\frac{1}{V}(\frac{\partial V}{\partial P})_T$
$= V(\frac{\partial^2 F}{\partial V^2})_T$	$= -T(\frac{\partial^2 F}{\partial T^2})_V$	$= -T(\frac{\partial^2 G}{\partial T^2})_P$	$= -\frac{1}{V}(\frac{\partial^2 G}{\partial P^2})_T$